Forum Acusticum 2011 - Aalborg - Denmark

Analytical beamforming for spherical loudspeaker arrays

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Summary
Spherical loudspeaker arrays have been recently studied in applications requiring directional sound sources in three dimensional space. Directivity of sound radiation, or beamforming, was achieved by driving each loudspeaker unit independently, where the design of beamforming weights was typically achieved by numerical optimization with reference to a given desired beam pattern. This is in contrast to the methods already developed for microphone arrays in general and spherical microphone arrays in particular, where beamformer weights are designed to satisfy a wider range of objectives, related to directivity and robustness, for example. This paper presents the development of analytical, physical-model-based, optimal beamforming framework for spherical loudspeaker arrays, similar to the framework already developed for spherical microphone arrays, facilitating efficient beamforming in the spherical harmonics domain, with independent steering. In particular, it is shown that from a beamforming perspective, the spherical loudspeaker array is similar to the spherical microphone array with microphones arranged around a rigid sphere. Experimental investigation validates the theoretical framework of beamformer design.

PACS no. 43.60.Fg, 43.38.Md

1. Introduction

Spherical loudspeaker arrays have been recently studied for applications such as electro-acoustic music performance, synthesizing the radiation pattern of musical instruments [1, 2]. A physical model of the loudspeaker array has been developed [3, 4], as a rigid sphere with vibrating caps mounted on its surface, employing spherical harmonics to describe caps vibration and sound radiation [5]. The latter can be controlled by numerically designing array weights to achieve a desired direcitivity function [6]. Although useful in generating desired beam patterns, these methods posses shortcomings. Typically, beampattern matching is the sole design objective, and so robustness against noise and uncertainty is not guaranteed. Also, no simple steering of the beam pattern is available.

This paper presents a beamforming design framework for spherical loudspeaker arrays that overcomes the shortcoming presented above. In particular, an analytical model of the spherical loudspeaker array is presented, from which analytical beamformer design methods are developed, based on the theory already developed for spherical microphone arrays. Experimental investigation of beamforming with a real array verify the theoretical results.

2. The spherical loudspeaker array

A spherical source is first modeled as a rigid sphere of radius r_0 with L spherical caps, representing loudspeaker units, positioned on its surface at locations (θ_l, ϕ_l), each imposing a constant radial surface velocity of v_l, $l = 1, ..., L$, at the surface segment they cover [4, 7]. Here θ_l represent elevation angle, and ϕ_l represent azimuth angle in a spherical coordinate system [8]. The radial velocity of the sphere surface at wave number k, $u(k, r_0, \theta, \phi)$, is composed of contributions from all L caps. The spherical Fourier transform [8] of the radial velocity, $u_{nm}(k, r_0)$, has been shown to satisfy [4]:

$$u_{nm}(k, r_0) = g_n \sum_{l=1}^{L} v_l(k) [Y_n^m(\theta_l, \phi_l)]^* \quad (1)$$

with

$$g_n \equiv \frac{4\pi^2}{2n+1} \left[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha) \right], \quad (2)$$

and with $Y_n^m(\cdot, \cdot)$ the spherical harmonics of order n and degree m, $P_n(\cdot)$ the Legendre polynomial, and α the aperture angle of each spherical cap.

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Given the radial velocity over the sphere surface, the spherical Fourier transform of the sound pressure $p(k,r,\theta,\phi)$, denoted by $p_{nm}(k,r)$, can be written as [5]:

$$p_{nm}(k,r) = i\rho_0 c \frac{h_n(kr)}{h'_n(kr_0)} u_{nm}(k,r_0) \tag{3}$$

with c the speed of sound, ρ_0 air density, $i=\sqrt{-1}$, and $h_n(\cdot)$ and $h'_n(\cdot)$ are the spherical Hankel function of the first kind of order n, and it's derivative, respectively [8]. Although source control is achieved in practice through control over caps velocity, one can assume a direct control over u_{nm} at orders $n \leq N$, with good accuracy, assuming far-field radiation is of interest, $kr >> N$, and $(N+1)^2 \leq L$ [4].

3. beamforming with a spherical source

Beamforming with spherical sources is employed with the aim of controlling the directivity of the sound radiated from the source. This is achieved by weighting the source signal $s(k)$ with weights $w_l(k)$ before driving the caps velocity, or loudspeaker units in practice, such that

$$v_l(k) = w_l(k)s(k),\ l = 1, ..., L. \tag{4}$$

We can now write

$$w_{nm}(k) = g_n \sum_{l=1}^{L} w_l(k)[Y_n^m(\theta_l,\phi_l)]^* \tag{5}$$

with

$$u_{nm}(k,r_0) = s(k)w_{nm}(k). \tag{6}$$

p_{nm} can now be rewritten as:

$$p_{nm}(k,r) = i\rho_0 c s(k) \frac{h_n(kr)}{h'_n(kr_0)} w_{nm}(k,r_0), \tag{7}$$

with the pressure $p(k,r,\theta,\phi)$ calculated as in inverse spherical Fourier transform of $p_{nm}(k,r)$ [5]. Beamforming design requires the computation of weights, $w_{nm}(k)$ or $w_l(k)$, such that the radiated sound pressure maintains some given design criteria.

An efficient formulation for far-field beamforming is derived next. We first assume that a far-field beam pattern is desired, such that $kr >> N$, where N is the highest order controlled by the source. In this case the following large-argument approximation can be employed [5]:

$$h_n(kr) \approx (-i)^{n+1} \frac{e^{ikr}}{kr} \tag{8}$$

We also use the Wronskian relation for $h'_n(kr_0)$ [5], and further denoting

$$b_n(kr) \equiv i\rho_0 ckr^2(-i)^n \times \left[j_n(kr) - \frac{j'_n(kr)}{h'_n(kr)} h_n(kr) \right], \tag{9}$$

we can rewrite the far-field pressure in the spherical harmonics domain as:

$$p_{nm}(k,r) = \frac{e^{ikr}}{r} s(k) b_n(kr_0) w_{nm}(k,r_0). \tag{10}$$

A normalized directivity can be formulated by multiplying with a factor re^{-ikr} [5], and assuming a unit input signal $s(k)=1$, with a further simplification achieved by considering axis-symmetric beam patterns, in a way similar to spherical microphone array beamforming [9], by selecting weights as follows:

$$w_{nm}(k) = \frac{d_n(k)}{b_n(kr_0)} [Y_n^m(\theta_0,\phi_0)]^* \tag{11}$$

where $d_n(k)$ is the one-dimensional axis-symmetric beamforming weighting function, and (θ_0,ϕ_0) is the look direction, forming the axis of symmetry. Using the spherical harmonics addition theorem [8], the far-field directivity function can now be written as:

$$B(k,\Theta) = \sum_{n=0}^{N} d_n(k) \frac{2n+1}{4\pi} P_n(\cos\Theta) \tag{12}$$

where Θ is the angle between the look direction (θ_0,ϕ_0) and the direction of radiated sound. Eq. (12) representing the beam pattern for the spherical source is exactly the same as the beam pattern equation for spherical microphone arrays [9]. Results obtained for spherical microphone arrays can therefore be used directly for the spherical loudspeaker array.

4. Directivity index and robustness

Beamformer design typically involves achieving a desired directivity, while maintaining necessary robustness constraints [10]. A common measure for array performance is the directivity factor, which for the spherical array is given by [11]:

$$Q = \frac{\left|\sum_{n=0}^{N} d_n(k)(2n+1)\right|^2}{\sum_{n=0}^{N} |d_n(k)|^2 (2n+1)}. \tag{13}$$

The Directivity Index (DI) is now defined as $10\log_{10} Q$. Another important measure is array robustness, which is a measure of the system sensitivity to noise, errors, uncertainties and perturbations. A common measure of robustness is the white-noise gain

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(WNG), which for a spherical array has been shown to equal [11]:

$$\mathrm{WNG} = \frac{\left|\sum_{n=0}^{\infty} d_n(k)(2n+1)\right|^2}{\sum_{n=0}^{N} \frac{|d_n(k)|^2}{|b_n(kr_0)|^2}(2n+1)}. \quad (14)$$

One possible optimal beamforming design method aims to find the beamforming weights d_n that maximize the directivity factor of a given spherical loudspeaker array. Due to the similarity between spherical loudspeaker and microphone arrays presented above, it can be shown that the optimal weights in this case satisfy $d_n = 1$ [12]. In a similar manner, the weights d_n that maximize the WNG can also be computed, and can be shown to satisfy $d_n(k) = \frac{4\pi|b_n(kr_0)|^2}{\sum_{n=0}^{N}|b_n(kr_0)|^2(2n+1)}$. A range of other beam pattern design methods can be applied to the spherical loudspeaker array in a similar manner, see for example [12].

5. Experimental study

The aim of this section is to provide an experimental examination of the beamforming design method presented in this paper. The experimental system includes a spherical loudspeaker array of radius $r_0 = 0.15\,\mathrm{m}$, with 12 individual loudspeaker units mounted on it's surface. The loudspeaker array is designed and produced by the Institute of Technical Acoustics, Aachen university. A microphone attached to a rotating system is used to spatially sample the sound pressure radiated by the loudspeaker array. The microphone positions followed the Gaussian sampling scheme [9], with a total of 242 samples, positioned at a radius of $r = 0.57\,\mathrm{m}$, achieving a spherical harmonic order of $N = 10$ at the analysis sphere. Once the microphone is positioned in place, the impulse response between each loudspeaker unit and the microphone is measured using a linearly swept-sine signal, in the range $0 - 1500\,\mathrm{Hz}$. The experiment was performed at the anechoic chamber, acoustics laboratory, Ben-Gurion University of the Negev, having inner dimensions of 2 m, certified as anechoic from 300 Hz.

Beamforming weights d_n were designed as described above, based on the maximum WNG method, and the maximum directivity method. Figure 1 shows balloon plots of the simulated and measured beam patterns for a design frequency of 400 Hz, using the maximum WNG method. Figure 2 shows a cross-section along the azimuth angle θ for elevation angle $\phi = \pi/2$. The figures show a reasonable similarity between simulated and measured beam patterns, validating the proposed design framework.

Figures 3 and 4 show similar results for 1000 Hz, using the maximum directivity design method, with a different beam pattern, which has a narrower main lobe. The measured beam pattern is similar to the simulated one, once again validating the design framework.

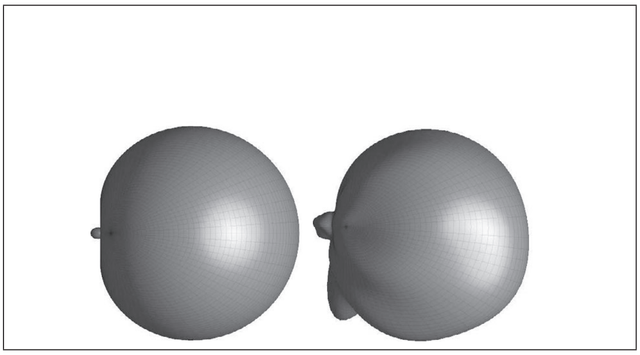

Figure 1: Balloon plot of the directivity function, designed using the maximum WNG method with $N = 2$, for an operating frequency of 400 Hz. Left: simulated, right: measured.

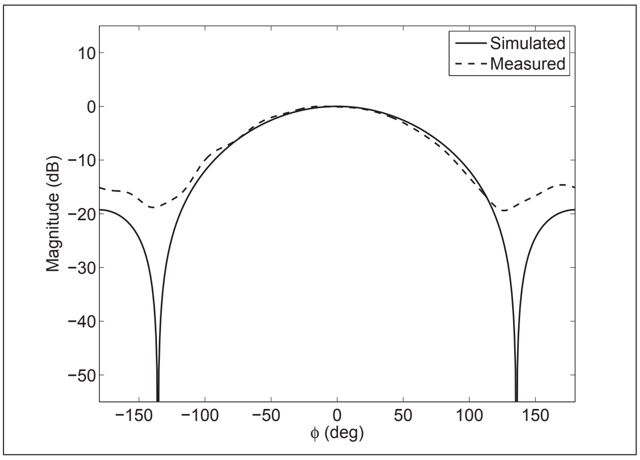

Figure 2: Same as in Fig. 1, but showing a cross-section along $(\pi/2, \phi)$.

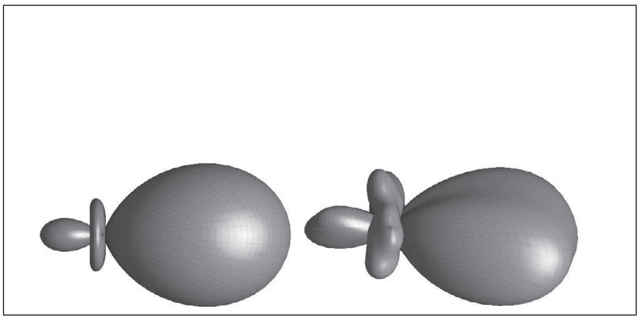

Figure 3: Balloon plot of the directivity function, designed using the maximum directivity method with $N = 2$, for an operating frequency of 1000 Hz. Left: simulated, right: measured.

6. CONCLUSIONS

This paper presented an efficient beamforming framework for spherical loudspeaker arrays, facilitating optimal, closed-form beam pattern design, with independent steering. The paper derives beamforming equations for the spherical loudspeaker array, showing similarity to spherical microphone arrays configured around a rigid sphere. This similarity facilitates the

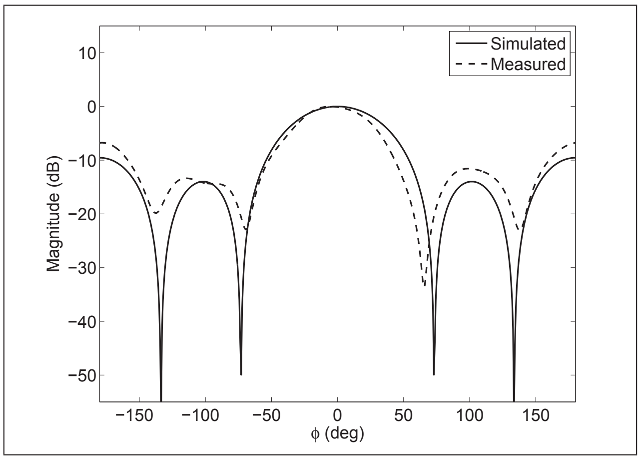

Figure 4: Same as in Fig. 3, but showing a cross-section along $(\pi/2, \phi)$.

use of a wide range of beamforming methods already developed for spherical microphone arrays. The design framework is then employed for beamforming with an experimental spherical loudspeaker array system, validating the theoretical results. The proposed framework can be used to produce directional radiation patterns with the spherical loudspeaker array in a wide range of applications.

Acknowledgement

This work was supported in part by the Ministry of Industry and Trade, grant no. 40161.

References

[1] O. Warusfel and N. Misdariis, "Directivity synthesis with a 3d array of loudspeakers: application for stage performance," in *Proceedings of the COSTG-6 Conference on Digital Audio Effects (DAFX-Ol), Limerick, Ireland*, 2001.

[2] R. Avizienis, A. Freed, P. Kassakian, and D. Wessel, "A compact 120 independent element spherical loudspeaker array with programmable radiation patterns," in *Proceedings of the 120th Audio Engineering Society Convention, Paris*, no. 6783, 2006.

[3] F. Zotter and R. Höldrich, "Modeling of radiation synthesis with spherical loudspeaker arrays," in *Proceedings of the 19th International Congress on Acoustics, Madrid*, no. COM-01-006, September 2007.

[4] B. Rafaely, "Spherical loudspeaker array for local active control of sound," *J. Acoust. Soc. Am.*, vol. 125, no. 5, pp. 3006–3017, May 2009.

[5] E. G. Williams, *Fourier acoustics: sound radiation and nearfield acoustical holography.* New York: Academic Press, 1999.

[6] A. M. Pasqual, J. R. de Franca Arruda, and P. Herzog, "Application of Acoustic Radiation Modes in the Directivity Control by a Spherical Loudspeaker Array," *Acta Acustica united with Acustica*, vol. 96, no. 1, pp. 32–42, 2010.

[7] F. Zotter and R. Höldrich, "Modeling a spherical loudspeaker system as a multipole source," in *Proceedings of the 33rd German Annual Conference on Acoustics, Stuttgart*, March 2007.

[8] G. Arfken and H. J. Weber, *Mathematical methods for physicists*, 5th ed. San Diego: Academic Press, 2001.

[9] B. Rafaely, "Analysis and design of spherical microphone arrays," *IEEE Trans. Speech Audio Proc.*, vol. 13, no. 1, pp. 135–143, January 2005.

[10] H. L. Van-Trees, *Optimum Array Processing (Detection, Estimation, and Modulation Theory, Part IV)*, 1st ed. Wiley-Interscience, 2002.

[11] B. Rafaely, "Phase-mode versus delay-and-sum spherical microphone array processing," *IEEE Sig. Proc. Let.*, vol. 12, no. 10, pp. 713–716, October 2005.

[12] B. Rafaely, Y. Peled, M. Agmon, D. Khaykin, and E. Fisher, "Spherical microphone array beamforming," in *Speech Processing in Modern Communications: challenges and perspectives*, I. Cohen, J. Benesty, and S. Gannot, Eds. Berlin: Springer-Verlag, 2010, ch. 11, pp. 281–305.

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