Additive Synthesis, Amplitude Modulation and Frequency Modulation

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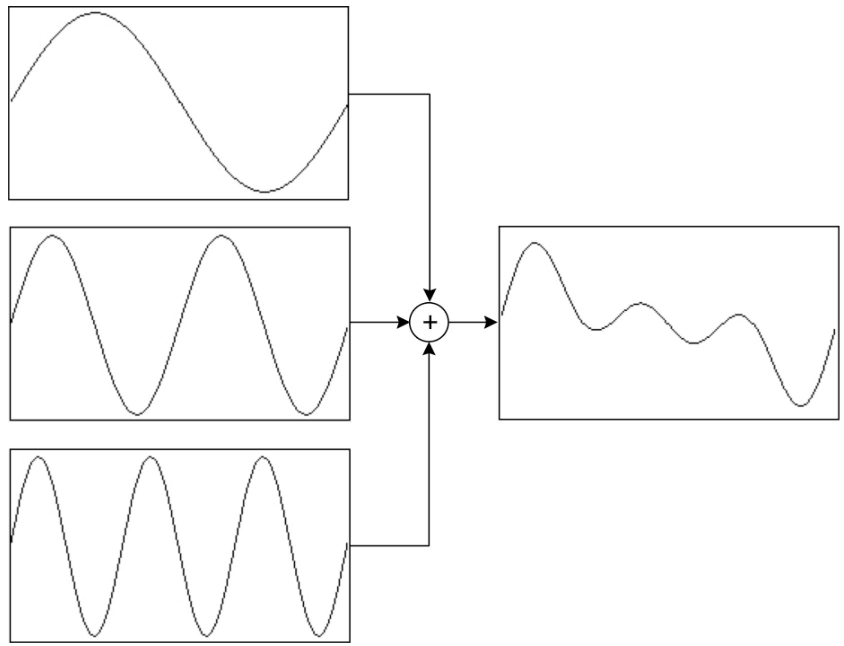
- Topics:

- Additive Synthesis
- Amplitude Modulation (and Ring Modulation)
- Frequency Modulation

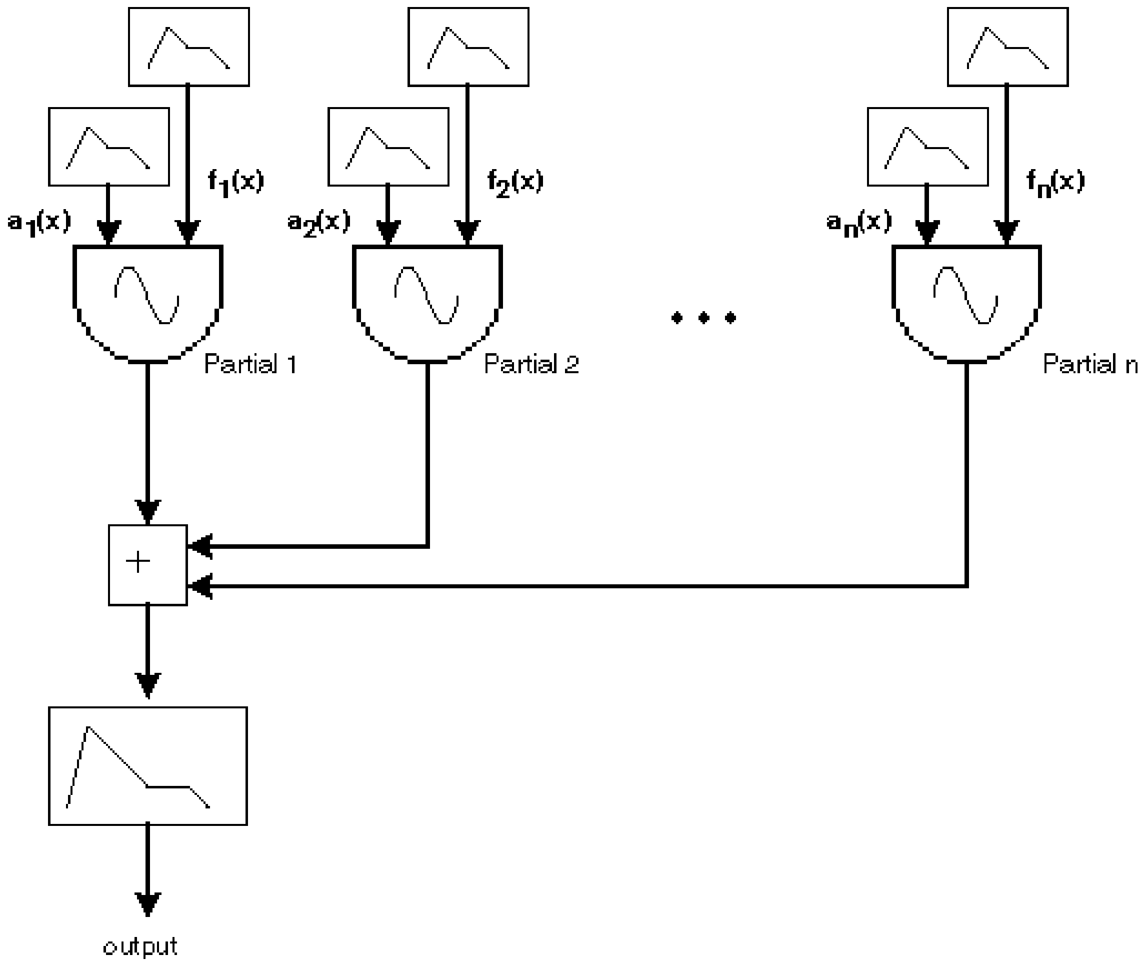
Additive Synthesis

• The technique assumes that any periodic waveform can be modelled as a sum sinusoids at various **amplitude envelopes** and **time-varying frequencies**.

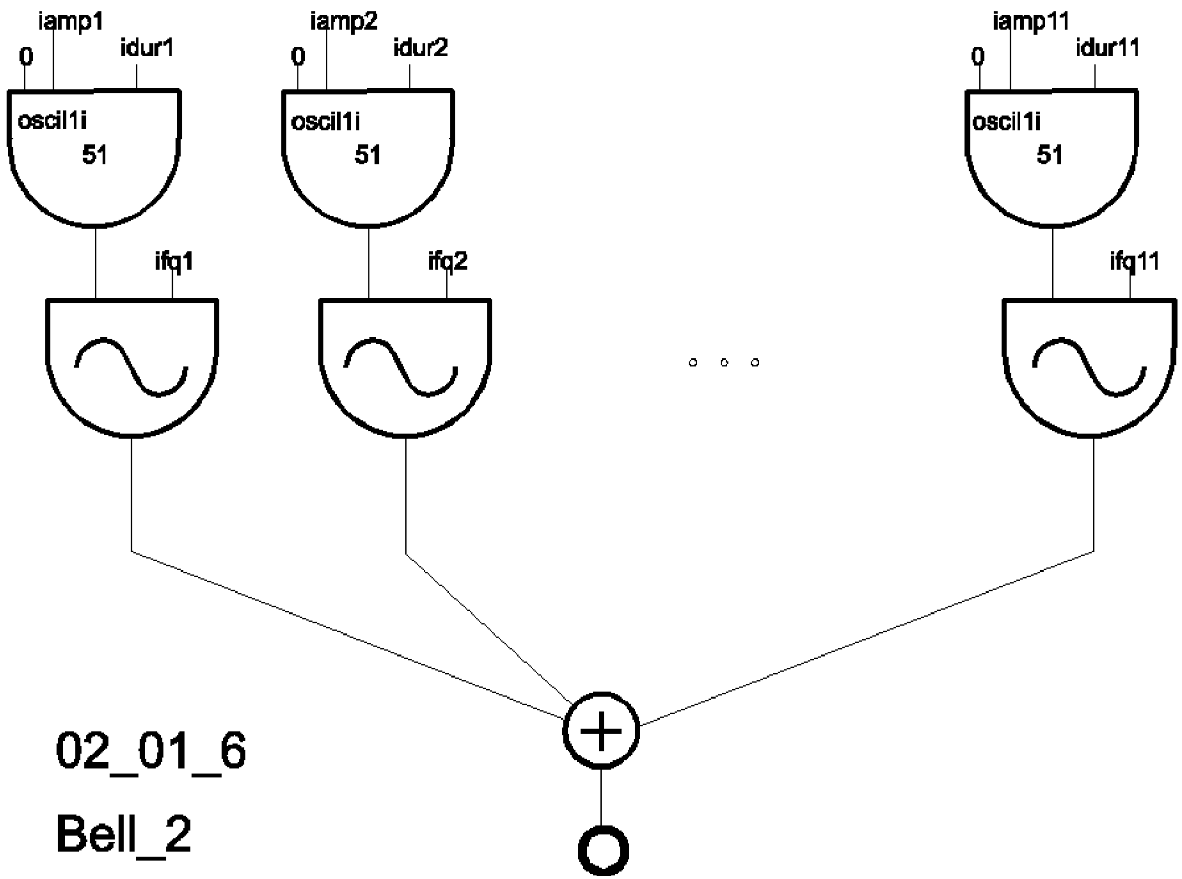
• Works by summing up individually generated sinusoids in order to form a specific sound.



Additive Synthesis



Additive Synthesis



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Bell_2

- A very powerful and flexible technique.

- But it is difficult to control manually and is computationally expensive.

- Musical timbres: composed of dozens of time-varying partials.

- It requires dozens of oscillators, noise generators and envelopes to obtain convincing simulations of acoustic sounds.

- The specification and control of the parameter values for these components are difficult and time consuming.

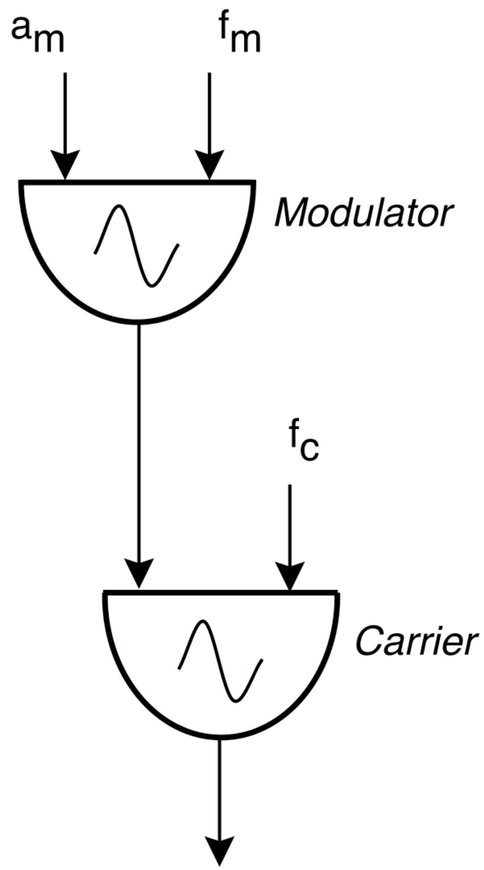
- Alternative approach: tools to obtain the synthesis parameters automatically from the analysis of the spectrum of sampled sounds.

Amplitude Modulation

• Modulation occurs when some aspect of an audio signal (***carrier***) varies according to the behaviour of another signal (***modulator***).

• AM = when a modulator drives the amplitude of a carrier.

• Simple AM: uses only 2 sinewave oscillators.



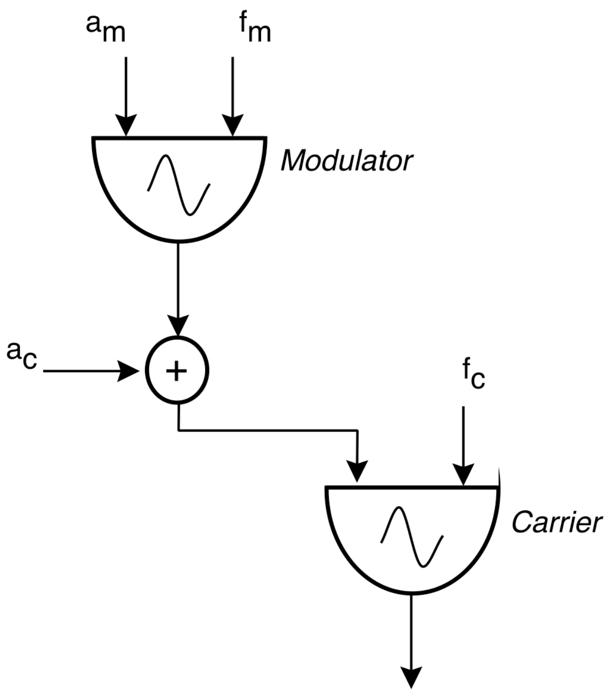
- Complex AM: may involve more than 2 signals; or signals other than sinewaves may be employed as carriers and/or modulators.

- Two types of AM:
    - a) Classic AM
    - b) Ring Modulation

Classic AM

- The output from the modulator is added to an offset amplitude value.

- If there is no modulation, then the amplitude of the carrier will be equal to the offset.

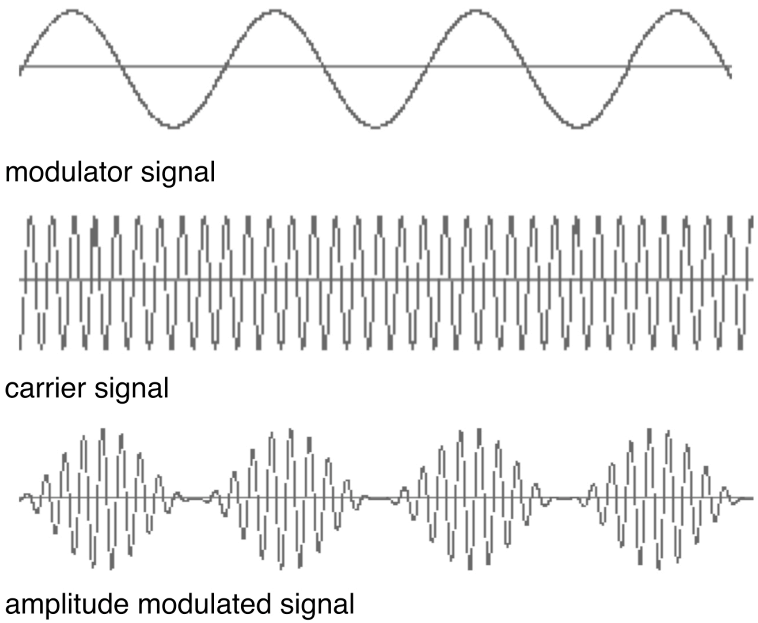


$$a_m = a_c \times mi$$

$$mi = \frac{a_c}{a_m}$$

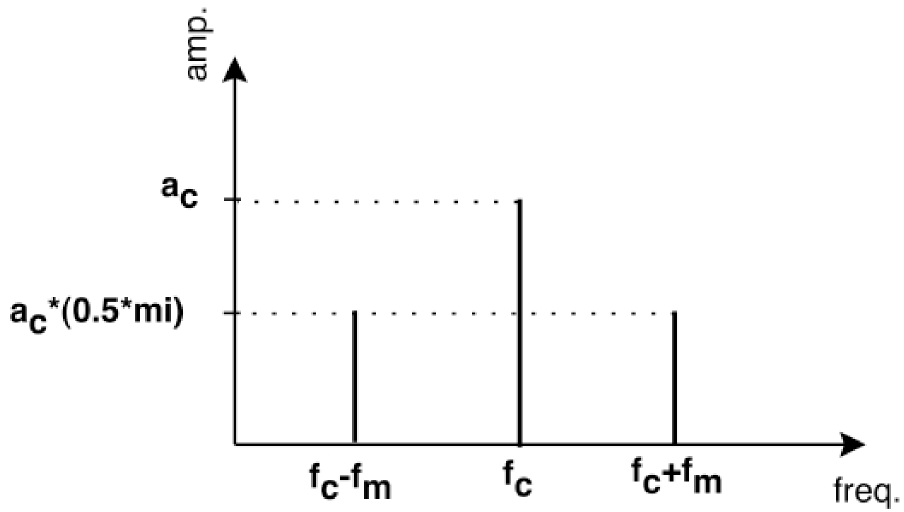
• If the modulation index is equal to zero, then there is no modulation.

• If it is higher than zero then the carrier will take an envelope with a sinusoidal variation.



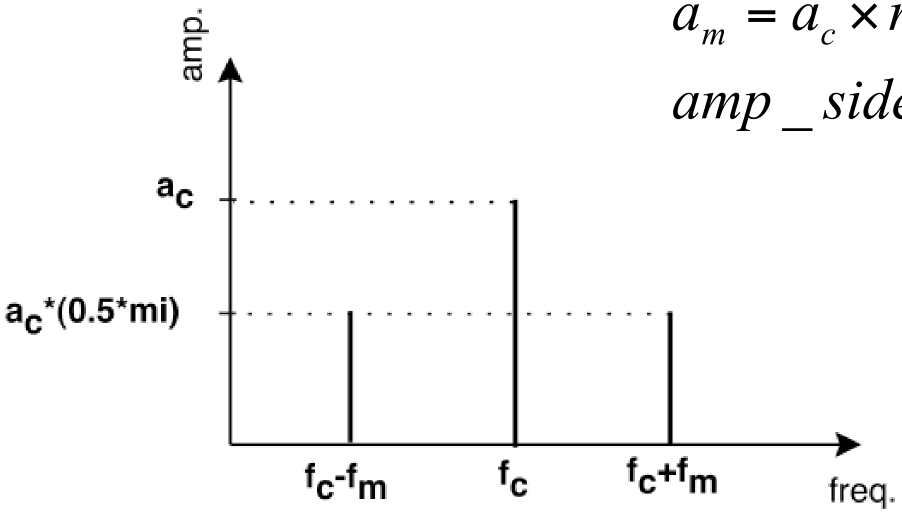
• In classic simple AM, the spectrum of the output contains 3 partials: at the frequency of the carrier + two sidebands, one below and one above the carrier's frequency value.

• Sidebands = **subtract** the frequency of the modulator from the carrier and **add** the frequency of the modulator to the carrier.



- Amplitudes

  - The carrier frequency remains unchanged
  - The sidebands are calculated by multiplying the amplitude of the carrier by half of the value of the modulation index, E.g. is *mi* = 1, the sidebands will have 50% of the amplitude of the carrier.

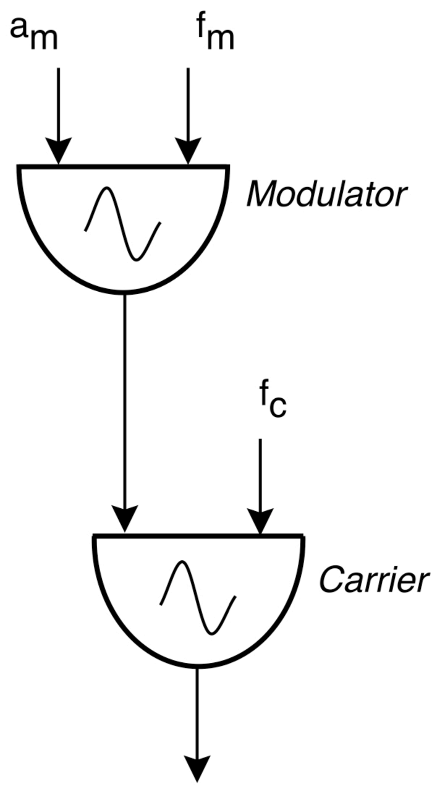


$$a_m = a_c \times mi$$

$$amp\_sidebands = a_c \times (0.5 \times mi)$$

Ring Modulation

- The amplitude of the carrier is determined entirely by the modulator signal.
- If there is no modulation, then there is no sound



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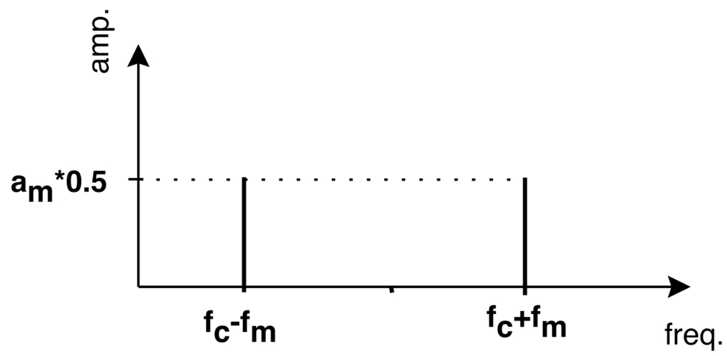
• When both signals are sinewaves, the resulting spectrum contains energy only at the sidebands.

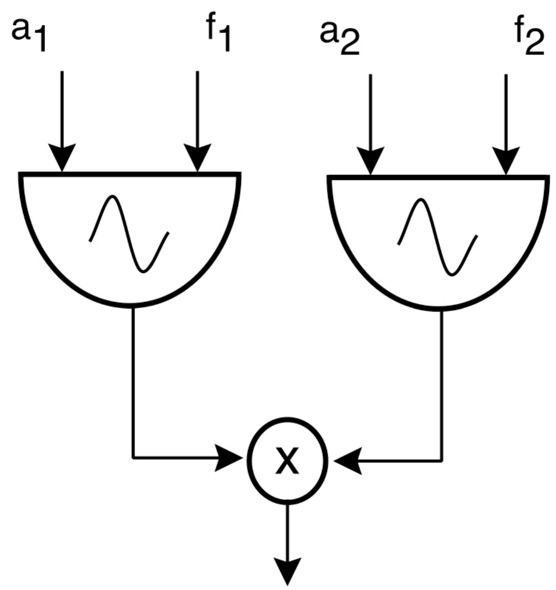
• The energy of the modulator is split between the 2 sidebands.

• The frequency of the carrier is not present.

• RM distorts the pitch of the signal; original pitch is lost.



- The multiplication of 2 signals is also a form of RM.



- Both classic AM and RM can use signals other than sinusoids, applying the same principles.

- Great care must be taken in order to avoid aliasing distortion (above 50% of the sampling rate).

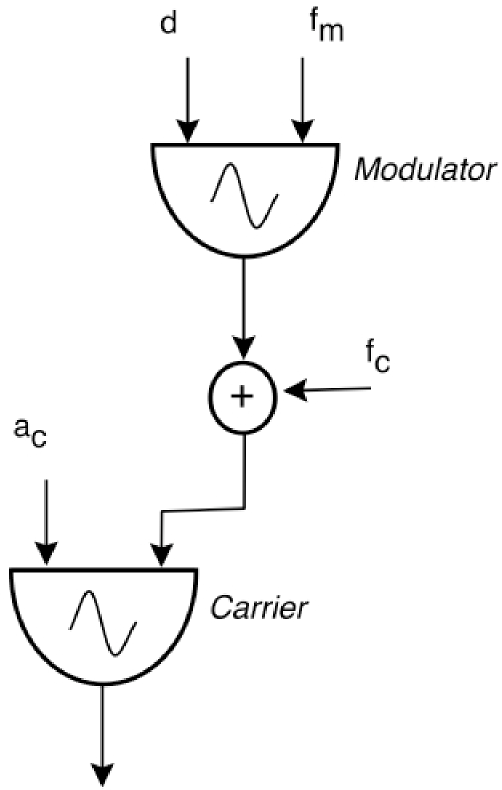
Frequency Modulation

• Modulation occurs when some aspect of an audio signal (***carrier***) varies according to the behaviour of another signal (***modulator***).

• FM = when a modulator drives the frequency of a carrier.

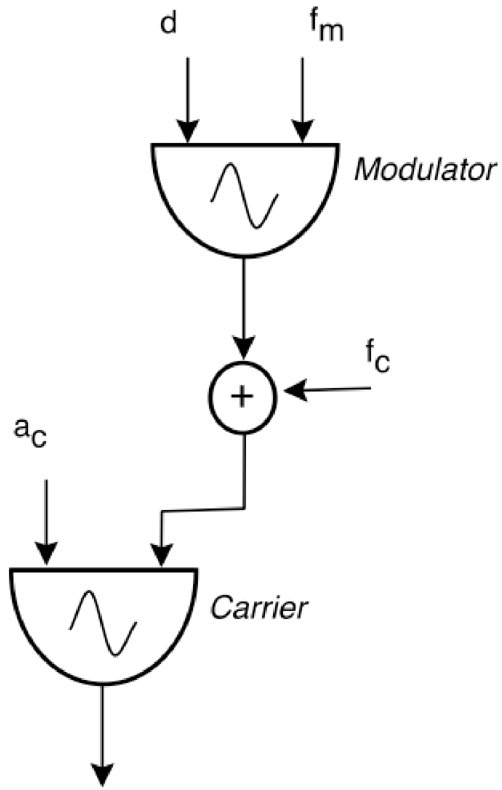
• Vibrato effect, good example to illustrate the principle of FM, with the difference
that vibrato uses sub-audio as the modulator (below 20 Hz).

• Simple FM: uses only 2 sinewave oscillators.



Simple FM

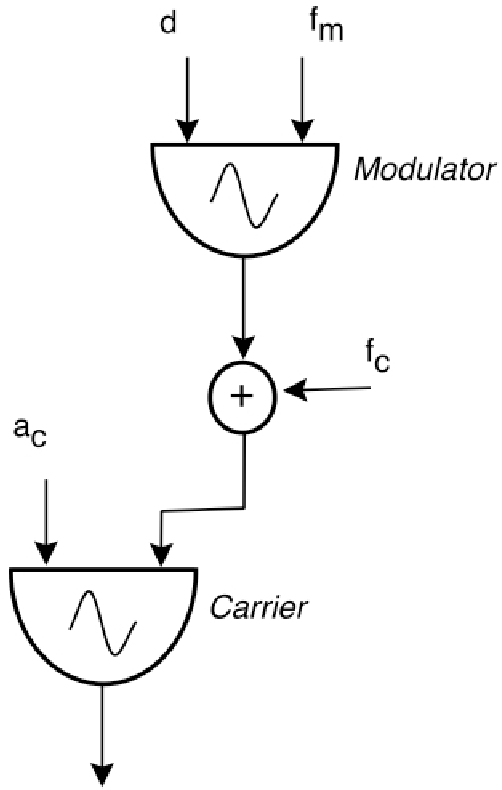
- The output of the modulator is offset by a constant, represented as f_c.
- If the amplitude of the modulator is equal to zero, then there is no modulation.
- In this case the output of the carrier will be a simple sinewave at frequency f_c.
- In the amplitude of the modulator is greater than zero, then modulation occurs.
- The output from the carrier will be a signal whose frequency deviates proportionally to the amplitude of the modulator.



• The "amplitude of the modulator" is called *frequency deviation*, and is represented as d.

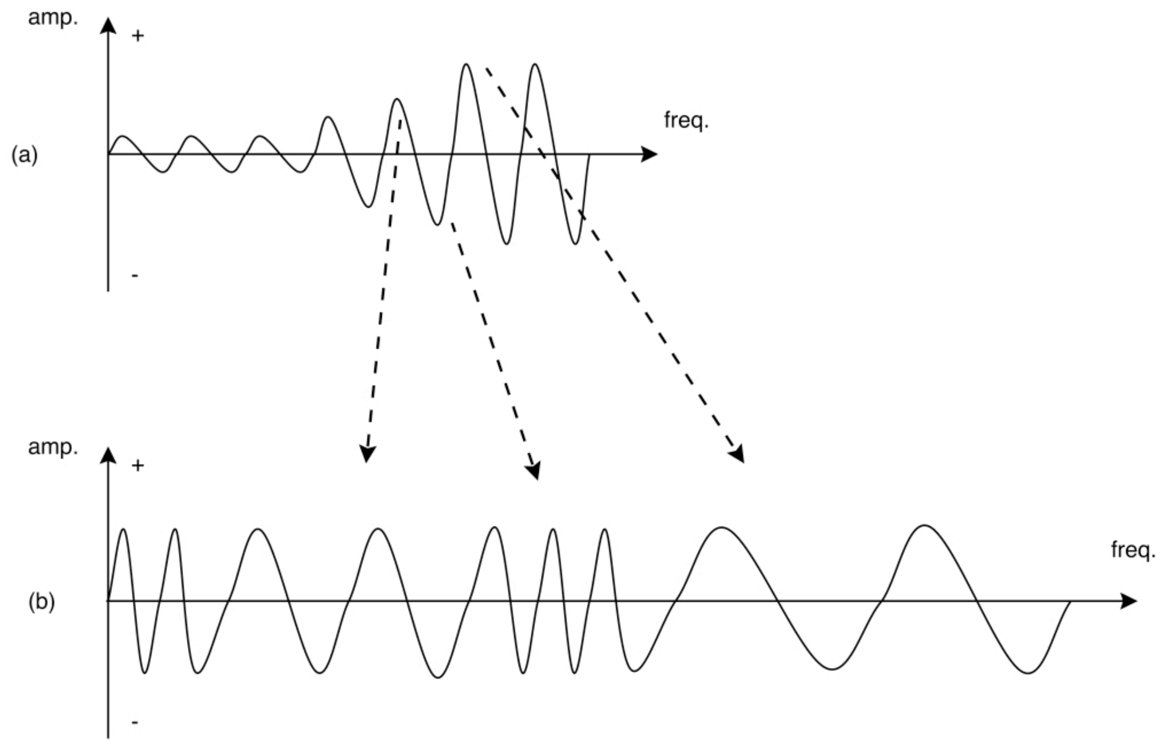
• The parameters of the simple FM algorithm are:

Frequency deviation = d
Modulator frequency = f_m
Carrier amplitude = a_c
Offset carrier frequency = f_c



• If f_m is kept constant whilst increasing d, then the period of the carrier's output will increasingly expand and contract proportionally to d.

• If d is kept constant whilst increasing f_m, then the rate of the deviation will become faster.



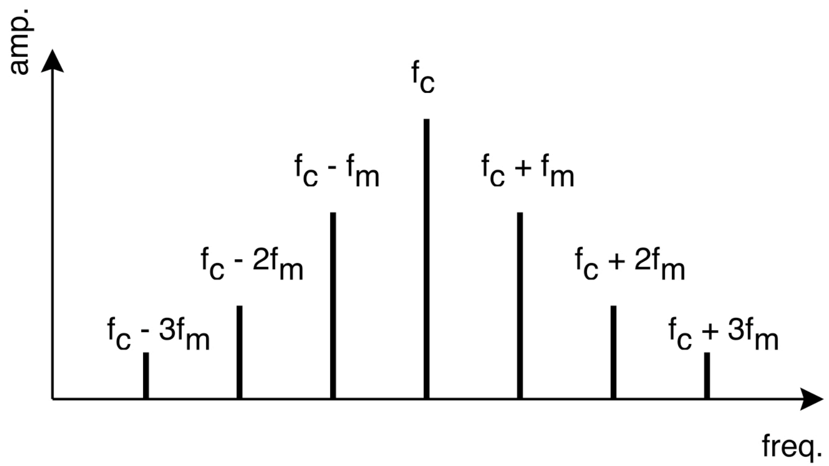
The spectrum of simple FM sounds

• The spectrum is composed of the carrier frequency (f_c) and a number of partials (called *sidebands*) on either side of it, spaced at a distance equal to the modulator frequency (f_m).

• The sideband pairs are calculated as follows, where k is an integer, greater than zero, which corresponds to the order to the partial counting from f_c:

$$f_c + k \times f_m$$

$$f_c - k \times f_m$$

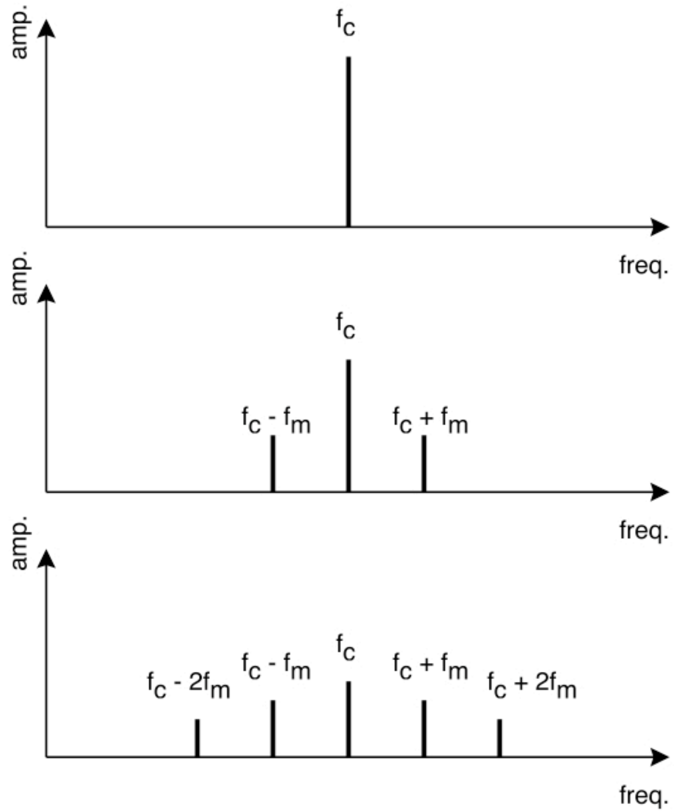


• The amplitude of the partials are determined mostly by the frequency deviation (*d*).

• If *d* = 0 then the power of the signal resides entirely in the offset carrier frequency (f_c).

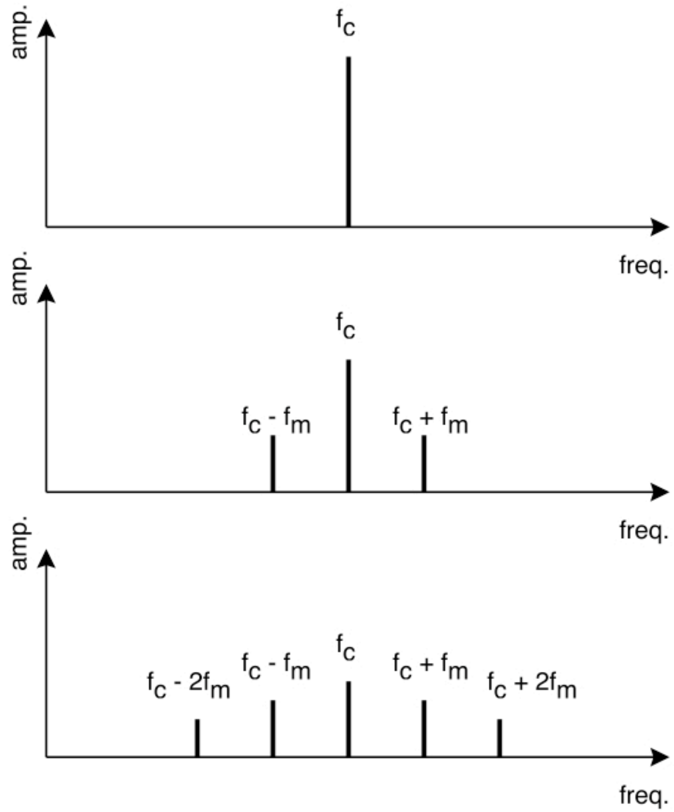
• Increasing the value of *d* produces sidebands at the expense of the power in f_c.

• The greater the value of *d*, the greater the number of generated partials and the wider the distribution of power between the sidebands



• Modulation index helps to control the number of audible sidebands and their respective amplitudes:

$$i = \frac{d}{f_m} \qquad d = i \times f_m$$



• As *i* increases from zero, the number of audible partials also increases and the energy of f_c is distributed among them.

• The number of sideband pairs with significant amplitude can generally be predicted as $i = 1$.

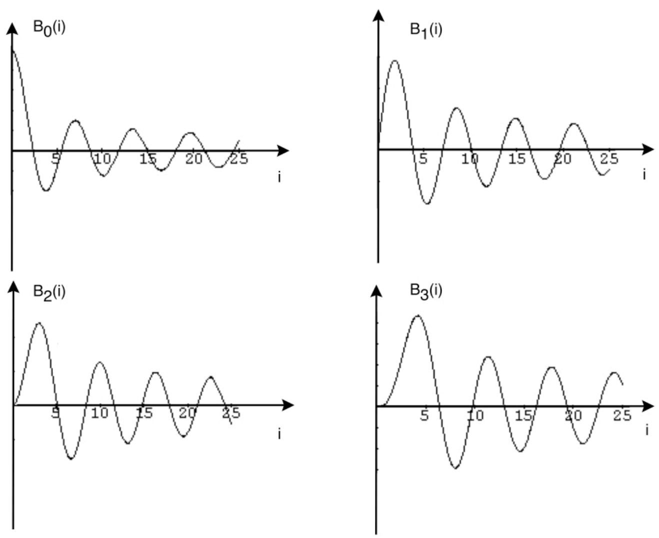
• Example if $i = 3$ then there will be 4 pairs of sidebands surrounding f_c.

Estimating the amplitude of the partials

• f_c “may” often be the most prominent partial in an FM sound; in this case it defines the pitch.

• The amplitudes of the partials are defined by a set of functions: *Bessel functions*.

• They determine **scaling factors** for pairs of sidebands, according to their position relative to f_c.

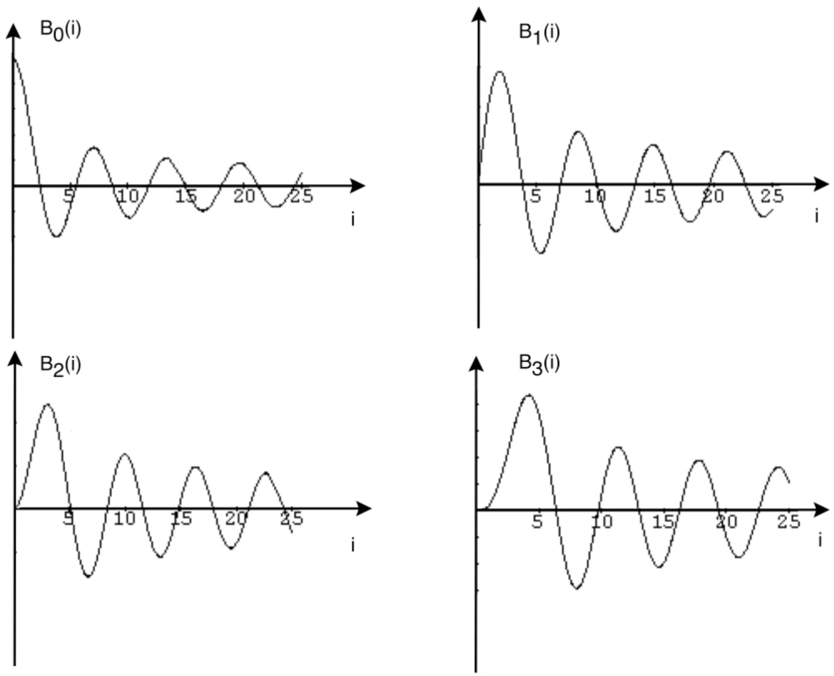


Bessel functions

- a_c usually defines the overall loudness of the sound

- The amplitudes of the partials are calculated by scaling ac according to the Bessel functions.

- Example: $B_0(i)$ gives the scaling for f_c, $B_1(i)$ for the first pair of sidebands (k=1), $B_2(i)$ for the second pair (k=2), $B_3(i)$ for the third (k=3), and so on.



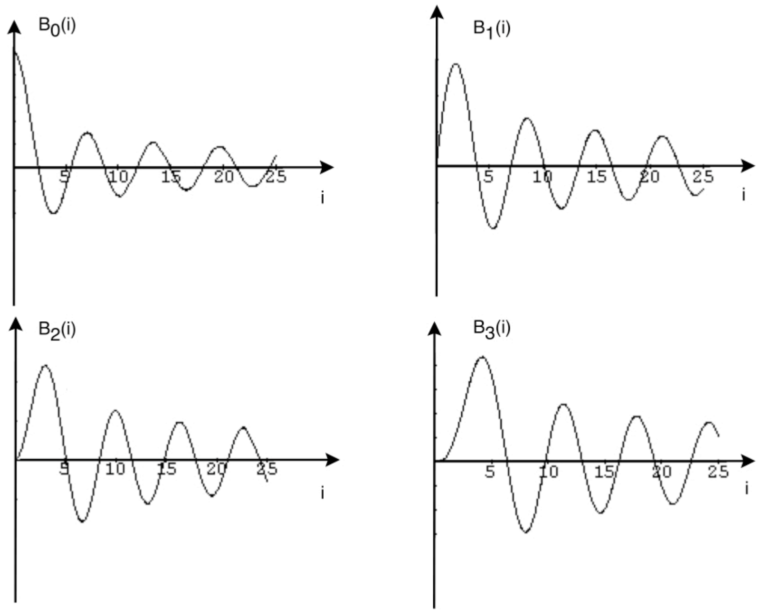
Bessel functions

• The vertical axis is the amplitude of scaling factor according to the value of i (mod. index) represented by the horizontal axis.

<u>Example:</u>

if i = 0 then f_c = max factor and all sidebands = 0

[$B_0(0) = 1$, $B_1(0) = 0$, $B_2(0) = 0$, $B_3(0) = 0$, etc.]



$$B_N(i)$$

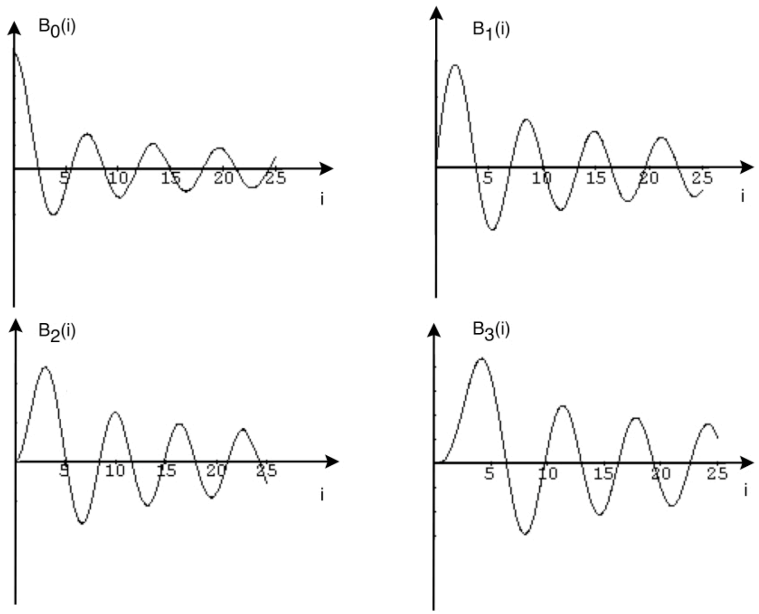
$$i = \frac{d}{f_m}$$

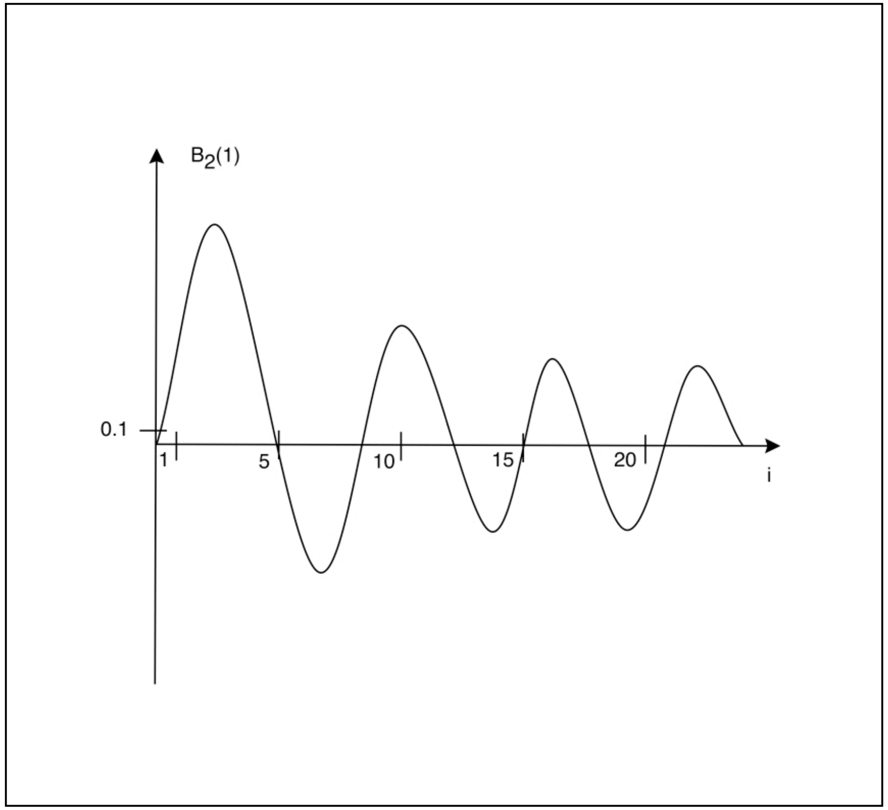
$$N = \text{sideband pair}$$

Example:

if i = 1 then fc = 0.76, 1st pair of sidebands = 0.44, 2nd pair = 0.11, etc.

[$B_0(0)$ = 0.76, $B_1(0)$ = 0.44, $B_2(0)$ = 0, $B_3(0)$ = 0.11, $B_4(1)$ = 0.01, etc.]



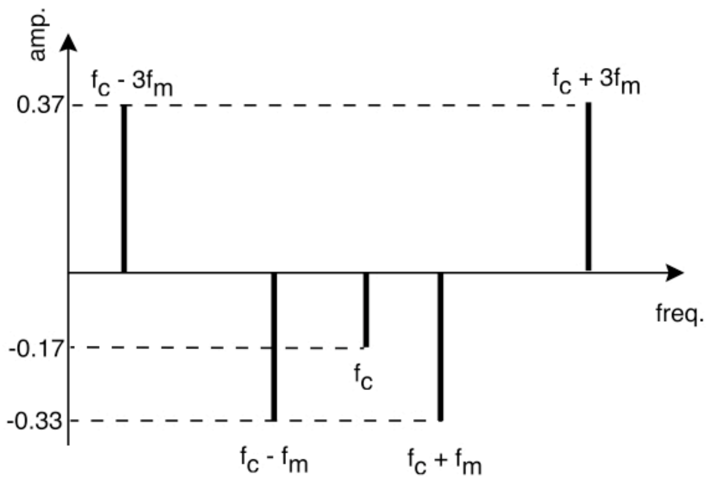


List of scaling factors (not exhaustive)

i	$B_0(i)$	$B_1(i)$	$B_2(i)$	$B_3(i)$	$B_4(i)$	$B_5(i)$	$B_6(i)$	$B_7(i)$	$B_8(i)$
0.0	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.938	0.242	0.030	0.002	0.000	0.000	0.000	0.000	0.000
1.0	0.765	0.440	0.115	0.019	0.002	0.000	0.000	0.000	0.000
1.5	0.512	0.558	0.232	0.060	0.011	0.001	0.000	0.000	0.000
2.0	0.223	0.576	0.352	0.129	0.034	0.007	0.001	0.000	0.000
2.5	–0.048	0.500	0.446	0.216	0.073	0.020	0.004	0.000	0.000
3.0	–0.260	0.340	0.486	0.309	0.132	0.043	0.011	0.002	0.000
3.5	–0.380	0.137	0.458	0.386	0.204	0.080	0.025	0.006	0.001
4.0	–0.400	–0.066	0.364	0.430	0.281	0.132	0.050	0.015	0.004
4.5	–0.032	–0.231	0.217	0.424	0.348	0.194	0.084	0.030	0.009
5.0	–0.177	–0.327	0.046	0.364	0.391	0.261	0.131	0.053	0.018
5.5	–0.006	–0.341	–0.117	0.256	0.396	0.320	0.186	0.086	0.033
6.0	0.150	–0.276	–0.242	0.115	0.357	0.362	0.245	0.130	0.056
6.5	0.260	–0.153	–0.307	–0.035	0.274	0.373	0.300	0.180	0.088
7.0	0.300	–0.004	–0.301	–0.167	0.157	0.347	0.340	0.233	0.128
7.5	0.266	0.135	–0.230	–0.258	0.023	0.283	0.354	0.283	0.174
8.0	0.171	0.234	–0.113	–0.291	–0.105	0.185	0.337	0.320	0.223
8.5	0.041	0.273	0.022	–0.262	–0.207	0.067	0.287	0.337	0.270
9.0	–0.090	0.245	0.145	–0.180	–0.265	–0.055	0.204	0.327	0.305
9.5	–0.194	0.161	0.228	–0.065	–0.270	–0.161	0.100	0.286	0.323
10.0	–0.245	0.043	0.254	0.058	0.220	–0.234	–0.014	0.216	0.317

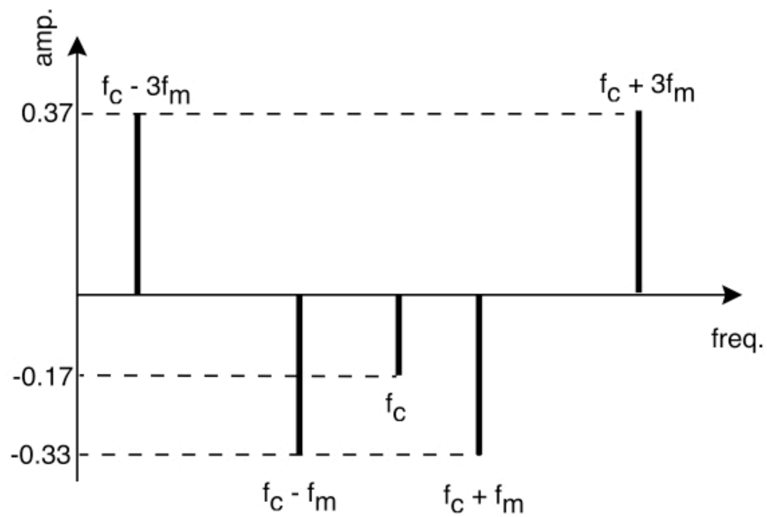
“Negative” amplitudes

- *The Bessel functions indicate that sidebands may have either positive or “negative” amplitude, depending on i.*
- *Example:*
  If i = 5, then 1^{st} pair of sidebands will be = -0.33
- *“Negative” amplitude does not exist: it only indicates that the sidebands are out of phase.*
- *Can be represented by plotting them downwards.*



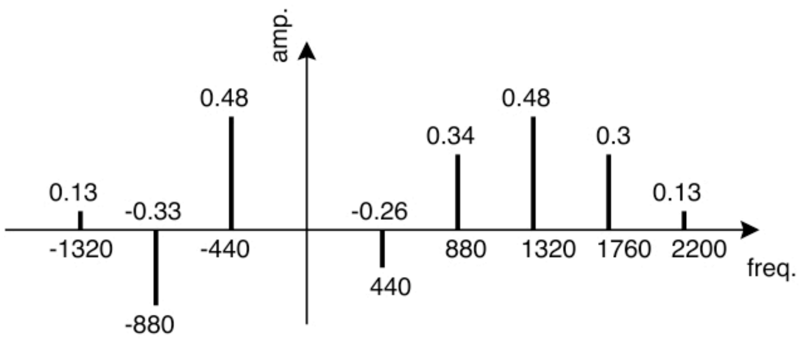
“Negative” amplitudes

- In general, the phase of the partials do not produce an audible effect…
- … Unless another partial of the same frequency happens to be present.
- In this case the amplitudes will either add or subtract, depending on their respective phases.



Negative frequencies & Nyquist distortion

- If f_c is too low and/or the *i* is too high, then the modulation produce sidebands that fall in the negative domain.
- As a rule, negative sidebands fold around the 0 Hz axis and mix with the others.
- Reflected sidebands will reverse their phase.



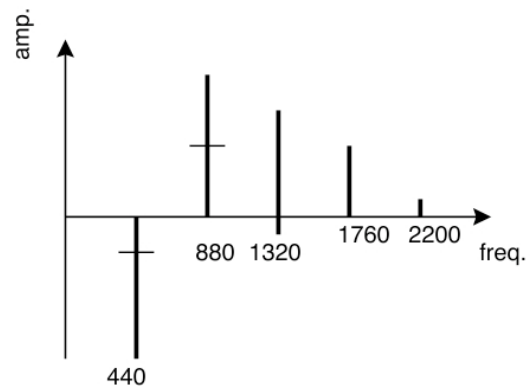
(a)

Negative frequencies

- Reflected sidebands will reverse their phase.

Example:

$$f_c = 440\text{Hz}, f_m = 440\text{Hz}, i = 3$$



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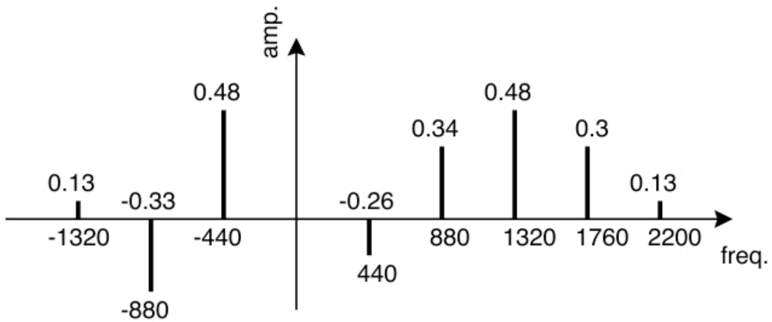

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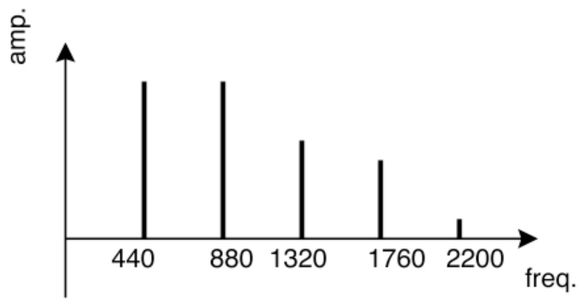


(c)

Nyquist distortion

- Partials falling beyond the Nyquist limit also fold over, and reflect into the lower portion of the spectrum.

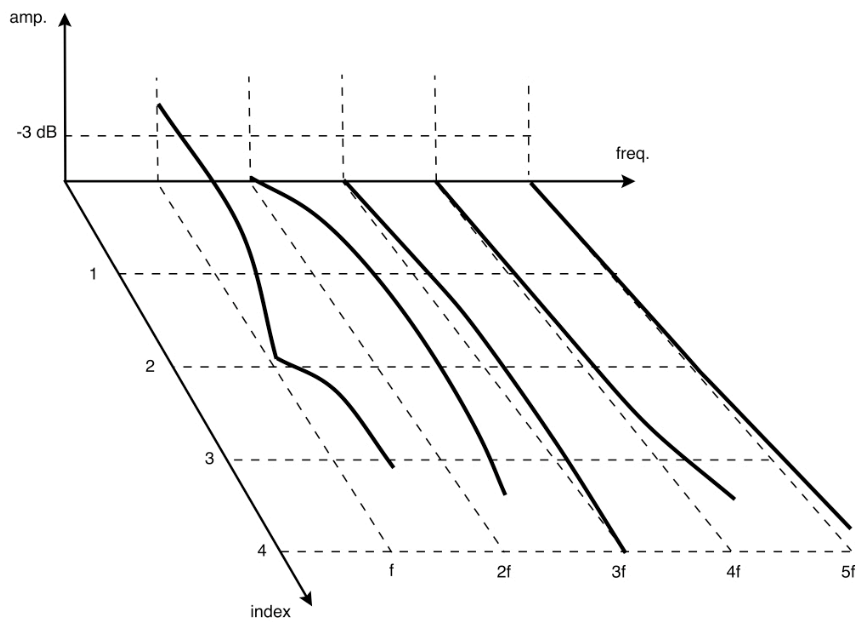
Synthesising time-varying spectra

- Modulation index *i* is an effective parameter to control spectral evolution.

- An envelope can be employed to time-vary *i* to produce interesting spectral envelopes that are unique to FM.

- A partial may increase or decrease its amplitude according to the slope the respective Bessel function.

- Linearly increasing I does not necessarily increase the amplitude of the high-order sidebands linearly.



Frequency ratios & sound design

- FM is governed by two simple ratios between FM parameters:

$$d : f_m = i \text{ (mod index)}$$

$$f_c : f_m = \text{frequency ratio}$$

- Freq ration is useful for achieving variations in pitch whilst maintaining the timbre virtually unchanged.

- If the freq ratio and the mod index if a simple FM instrument are maintained constant, but f_c is modified then the sounds will vary in pitch, but the timbre remains unchanged.

• It is more convenient to think of in terms of freq ratios rather than in terms of values for f_c and f_m.

$$d : f_m = i \text{ (mod index)}$$

$$f_c : f_m = \text{frequency ratio}$$

• It is clear to see that 220 : 440 are in ratio 1:2, but not so immediate for 465.96 : 931.92.

• As a rule of thumb, freq ratios should always be reduced to their simplest form. For example, 4:2, 3:1.5 and 15:7.5 are all equivalent to 2:1

FM directives in terms of simple ratios

Case 1: if f_c is equal to any integer and f_m is equal to 1, 2, 3 or 4, then the resulting timbre will have a distinctive pitch, because the offset carrier frequency will always be prominent. **FM6**

Case 2: if f_c is equal to any integer and f_m is equal to any integer higher than 4, then the modulation produces harmonic partials but the fundamental may not be prominent. **FM7**

Case 3: if f_c is equal to any integer and f_m is equal to 1, then the modulation produces a spectrum composed of harmonic partials; e.g. the ratio 1:1 produces a sawtooth-like wave. **FM8**

Case 4: if f_c is equal to any integer and f_m is equal to any even number, then the modulation produces a spectrum with some combination of odd harmonic partials; e.g. the ratio 2:1 produces a square-like wave. **FM9**

Case 5: if f_c is equal to any integer and f_m is equal to 3, then every third harmonic partial of the spectrum will be missing; e.g. the ratio 3:1 produces narrow pulse-like waves. **FM10**

Case 6: if f_c is equal to any integer and f_m is not equal to an integer, then the modulation produces non-harmonic partials; e.g. 2:1.29 produces a 'metallic' bell sound. **FM11**

Composite FM

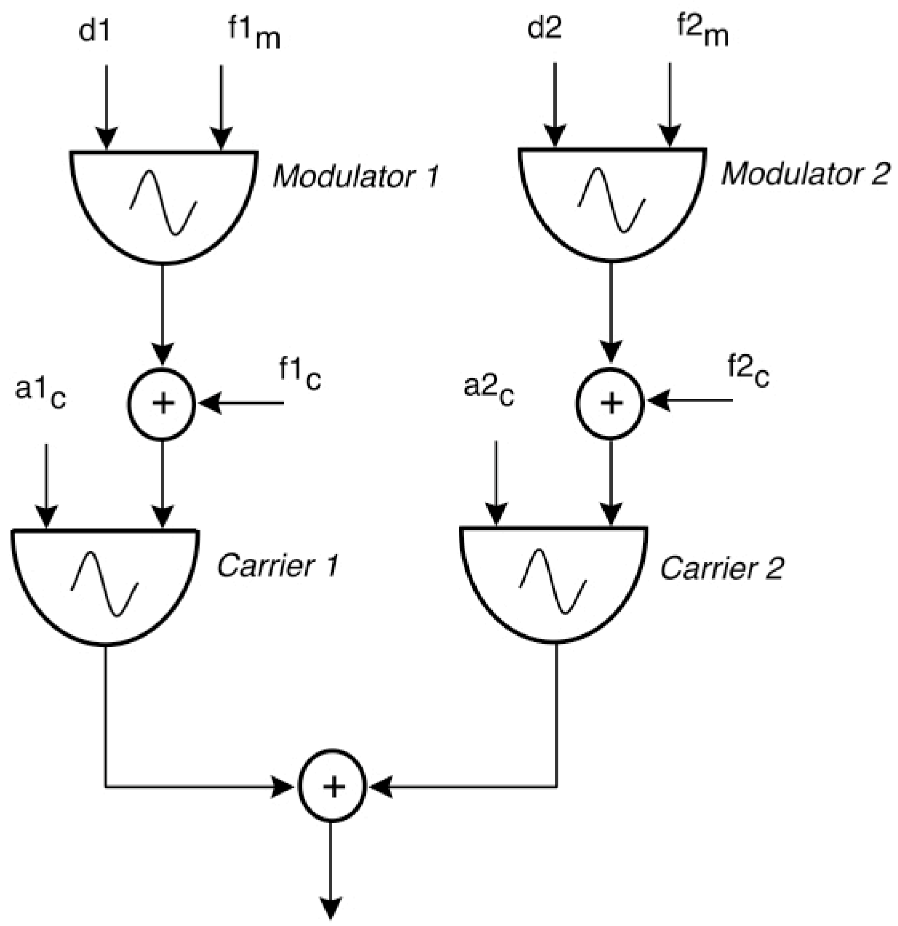
- Involves 2 or more carrier oscillators and/or 2 or more modulator oscillators.
- Produces more sidebands, but the complexity of the calculations for predict the spectrum also increases.
- Basic combinations:

  a) Additive carriers with independent modulators
  b) Additive carriers with one modulator
  c) Single carrier with parallel modulators
  d) Single carrier with serial modulators
  e) Self-modulating carrier

Additive carriers with independent modulators

- Composed of 2 or more simple FM instruments in parallel.

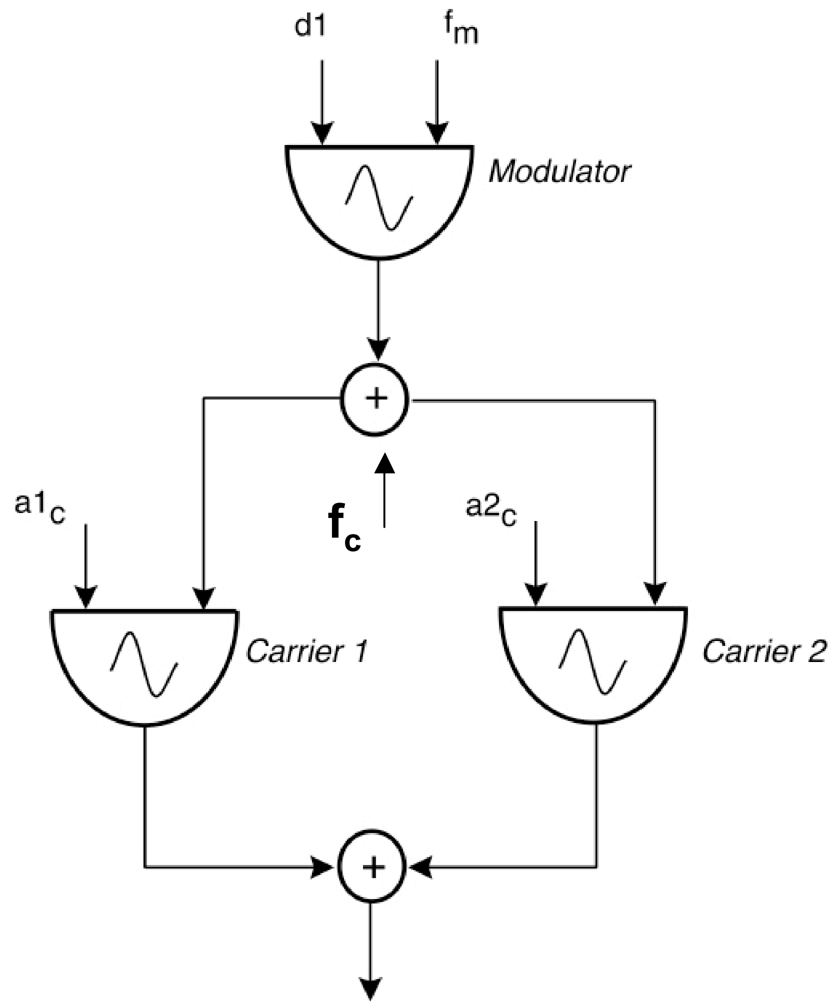
- The spectrum is the result of the addition of the outputs from each instrument.



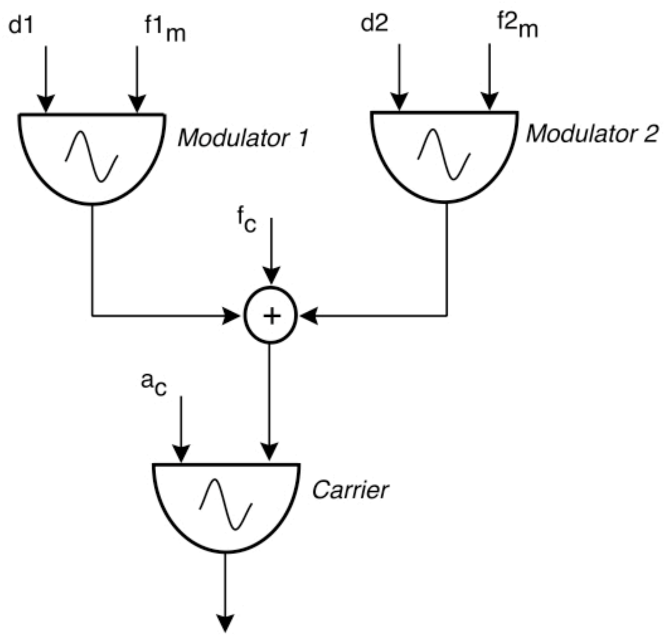
Additive carriers with 1 modulator

- One modulator oscillator modulates 2 or more oscillators.

- The spectrum is the result of the addition of the outputs from each carrier oscillator.



Single carrier with parallel modulators



- Modulator is the result of 2 or more sinewaves added together.

- The FM formula is expanded to accommodate multiple modulator freq (f_m) and mod indices (i).

- In the case of 2 parallel modulator the sideband pairs are calculated as follows:

$$f_c - (k_1 \times f_{m1}) + (k_2 \times f_{m2})$$
$$f_c - (k_1 \times f_{m1}) - (k_2 \times f_{m2})$$
$$f_c + (k_1 \times f_{m1}) + (k_2 \times f_{m2})$$
$$f_c + (k_1 \times f_{m1}) - (k_2 \times f_{m2})$$

$$f_c - (k_1 \times f_{m1}) + (k_2 \times f_{m2})$$
$$f_c - (k_1 \times f_{m1}) - (k_2 \times f_{m2})$$
$$f_c + (k_1 \times f_{m1}) + (k_2 \times f_{m2})$$
$$f_c + (k_1 \times f_{m1}) - (k_2 \times f_{m2})$$

- Each of the partials produced by one modulator oscillator (k_1 x f_{m1}) forges a “local carrier” for the other modulator oscillator (k_2 x f_{m2}) **.**

- The amplitude scaling factor result from the multiplication of the respective Bessel functions: $B_n(i_1)$ x $B_m(i_2)$.

Example: (see Appendix I of Computer Sound Design Book)

$$i_1 = 1.5$$

$$i_2 = 1.0$$

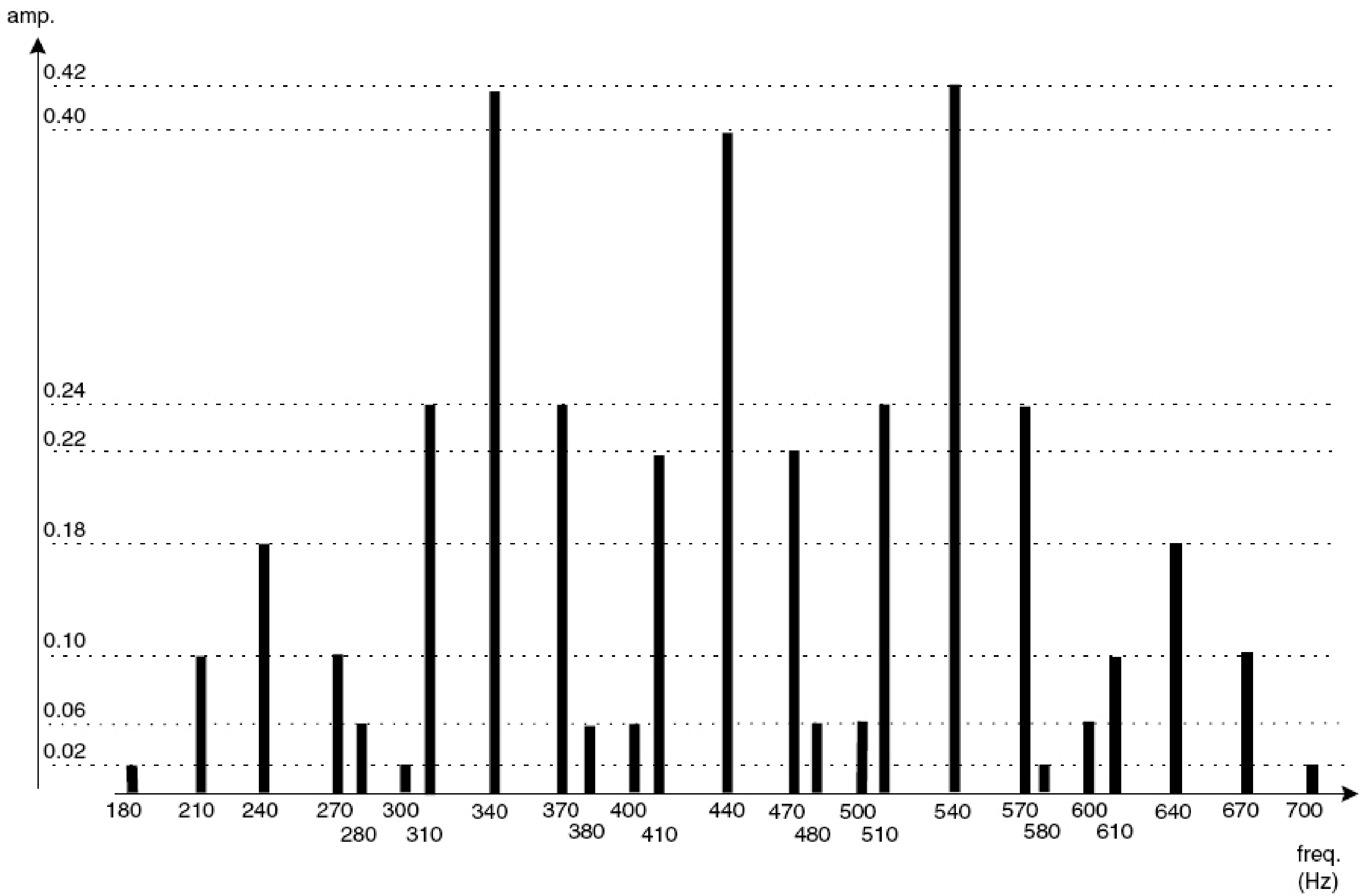
$$f_c = 440\,\text{Hz}$$

$$f_{m1} = 100\,\text{Hz}$$

$$f_{m2} = 30\,\text{Hz}$$

$$\begin{aligned} B_n(1.5) \times B_m(1.0) \text{ for } & 440 - (n \times 100) + (m \times 30) \\ & 440 - (n \times 100) - (m \times 30) \\ & 440 + (n \times 100) + (m \times 30) \\ & 440 + (n \times 100) - (m \times 30) \end{aligned}$$

where $n = \{0, 1, 2\}$ and $m = \{0, 1, 2\}$.


amp.
0.42
0.40
0.24
0.22
0.18
0.10
0.06
0.02
180
210
240
270
280
300
310
340
370
380
400
410
440
470
480
500
510
540
570
580
600
610
640
670
700
freq.
(Hz)

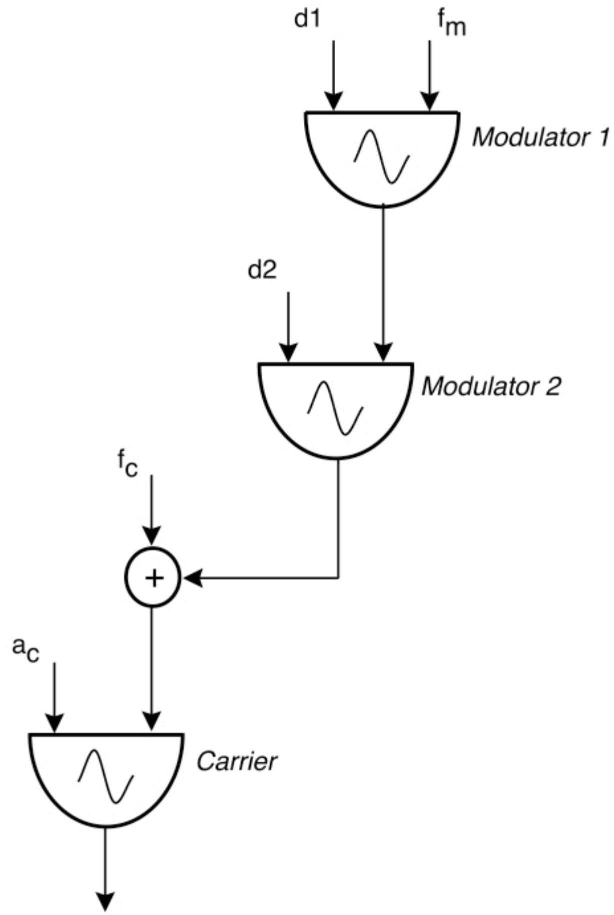
Single carrier with serial modulators

• The modulating signals is a frequency modulated signal.

• The sidebands are calculated using the same method as for parallel modulators, but the amplitude scaling factors is different:

• The order of the outermost modulator is used to scale the modulations index of the next modulator: $B_n(i_1)$ x $B_m(n \times i_2)$.

• Note: no sidebands from $B_m(i)$ are generated: $B_0(i_1)$ x $B_1(0 \times i_2) = 0$.



Further reading:

- Three Modelling Approaches to Sound Design, by E R Miranda (PDF file tutorial3.pdf)
- The Amsterdam Csound Catalogue: http://www.music.buffalo.edu/hiller/accci/